International Journal of Theoretical Physics, Vol. 22, No. 8, 1983

Finite Quantum Field Theory and Neutrinodynamics

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Received May 19, 1981

In quantum neutrinodynamics (photon–neutrino weak coupling) all the renormalization constants vanish and therefore the field equations cannot be expressed in terms of unrenormalized field quantities. This helps us to formulate quantum neutrinodynamics as a convergent quantum field theory. It is also pointed out that from the viewpoint of the unified model of weak and electromagnetic interaction as developed on the basis of the photon–neutrino weak coupling by Bandyopadhyay, quantum electrodynamics also manifests itself as a convergent field theory.

1. INTRODUCTION

The work of Tomonga, Schwinger, and Feynman led to a milestone in the development of quantum field theory. So far as the renormalization field theory is concerned we are accustomed to divergence difficulties, but we expect finite unambiguous results for experimentally observable quantities. However, in perturbation theory, the renormalization constants are infinite so that each calculation of a physical quantity has an infinity buried in it. Whether this infinity is a disease of the mathematical techniques of perturbation expansions, or whether it is a symptom of the ills accompanying the idealization of a continuum theory we do not know.

0020-7748/83/0800-0677$03.00/0

In recent times, there has arisen much interest in the formulation of a finite quantum field theory. From the auxiliary field concept Pauli and Villars (1949) tried to develop a finite and self-consistent field theory. But in any case this is absolutely an *ad hoc* manipulation of the theory. It is well known that the divergences in quantum electrodynamics (QED) associated with the unrenormalized theory can be lumped into the vertex, wave function, and coupling constant renormalization constants Z_1, Z_2, and Z_3 and δm, respectively. By a suitable choice of the gauge parameter (generalized Landau gauge) the gauge-dependent constants $Z_1 = Z_2$ are rendered finite. But it is to be emphasized that in quantum electrodynamics within the framework of the perturbation theory (QEDP), Z_3^{-1} diverges to all orders as shown by Gell-Mann and Low (1954). For Z_3 finite they imposed an eigenvalue condition

$$\psi(\alpha_0) = 0$$

and determined the asymptotic coupling α_0 rather than the physical coupling α. But the condition $\alpha < \alpha_0$, from the spectral function positivity, leaves α as a free parameter. This remarkable conclusion was reached in the subsequent work of Johnson and Baker (1969), who showed that if Z_3 is finite, the renormalization constant Z_2 and the bare mass of the electron m_0 can also be finite. Johnson, Baker, and Willey (1964, 1967) also studied photon propagators without photon self-energy insertions. By introducing a cutoff Λ^2 they defined the unrenormalized photon propagator and photon self-energy in the asymptotic region and finally derived the necessary condition for $Z_3 \neq 0$. Following the procedure of Coleman and Jackiw (1971), Adler and Bardeen (1971) derived the Callan–Symanzik equation (Callan, 1970; Symanzik, 1970) and showed that the unrenormalized m_0, z_2, and $\pi(k^2)$ become finite by introducing an ultraviolet and infrared cutoffs by Λ^2 and μ^2, respectively. Now the short-distance behavior of the photon propagator is considered within the context of the corresponding Callan–Symanzik equation. Maris and Dillenburg (1970) studied the connection between the field theory and the perturbation expansion procedure of QED, considering the bare spinor mass and Z_3 equal to zero. The theory does not contain any constant of Nature and is dilatational and gauge invariant, both invariances being spontaneously broken. The vanishing of the bare spinor mass is a necessary condition for the mass normalization to be finite and the resulting dilatational invariance of the theory is attractive from the general point of view.

In this paper, we shall show that quantum neutrinodynamics (QND) incorporating the photon–neutrino weak coupling as suggested by Bandyopadhyay (1968) is indeed inherently convergent, and suggests the introduction of a fundamental length in Nature. Again, if we take into account the unified picture of weak and electromagnetic interaction on the basis of the dynamical origin of charge and mass of the electron (or muon), and the consequent space-time quantization as suggested in earlier papers (Bandyopadhyay, 1973; 1974), we can have an electrodynamics where the fundamental interaction is the photon–neutrino weak coupling so that QED becomes a convergent theory (Bandyopadhyay and Roy, 1976).

2. FINITE QUANTUM FIELD THEORY: NEUTRINODYNAMICS

In the recent past, Bandyopadhyay (1974) showed that the photon–neutrino weak coupling theory has certain characteristic features. In fact, the theory is "renormalizable" in the conventional sense and there is no unitarity catastrophe in the high-energy processes. Moreover, this helps to represent the photon as a composite state of neutrino–antineutrino pair both from the field theoretic (Bandyopadhyay and Raychaudhuri, 1971) and Bethe–Salpeter formalism (Sarkar et al., 1975). The fundamental criterion for this compositeness condition $Z_3 = 0$ is found to be uniquely satisfied in neutrinodynamics (Sarkar et al., 1975). Again, because of the conservation of the two-component neutrino current, the Ward–Takahasi identity holds and as a result we have $Z_1 = Z_2$. Maris and Dillenburg (1970) have studied QED, taking the vanishing bare spinor mass and $Z_3 = 0$. They have shown that in this special case $Z_1 = Z_2 = 0$ in all physical gauges. Evidently this result is valid in our present case too and thus in neutrinodynamics we get the interesting result that all the renormalization constants become zero. We write here explicitly the renormalized quantities:

$$\phi = Z_2^{-1/2}\phi^{(u)} \tag{1}$$

$$\begin{aligned} j_\mu &= Z_1\bar{\phi}_\nu\gamma_\mu\phi_\nu \\ &= Z_1 Z_2^{-1}\bar{\phi}_\nu^{(u)}\gamma_\mu\phi_\nu^{(u)} \end{aligned} \tag{2}$$

$$A_\mu = Z_3^{-1/2}A_\mu^{(u)} \tag{3}$$

$$g = Z_1^{-1}Z_2 Z_3^{1/2} g^{(u)} \tag{4}$$

where ϕ is the two-component neutrino wave functions and the superscript (u) stands for the unrenormalized value.

For completeness, we recapitulate here the derivation of the $Z_3 = 0$ condition in neutrinodynamics as discussed in Bandyopadhyay and

Raychaudhuri (1971). It is to be remarked here that photons can interact weakly only with massless two-component neutrinos (Bandyopadhyay, 1968) having an interaction lagrangian of the form

$$L_I = ig\bar{\phi}\gamma_\mu\phi A_\mu$$
$$= ig\bar{\psi}\gamma_\mu(1+\gamma_5)\psi A_\mu$$

Here ϕ is the two-component neutrino wave function, ψ is the four-component function defined as

$$\phi = \tfrac{1}{2}(1+\gamma_5)\psi$$

and g is the photon–neutrino weak coupling constant. Analyzing QEDP in a similar way, we can use the following spectral representation to calculate Z_3^{-1} for neutrinodynamics:

$$Z_3^{-1} = \int_0^\infty d\sigma^2 \rho_R(\sigma^2) \tag{5}$$

where

$$\rho_R(\sigma^2) = \delta(\sigma^2) + \frac{g^2}{12\pi^2}\theta(\sigma^2)\cdot\frac{1}{\sigma^2}$$

with

$$\theta(\sigma^2) = 1, \qquad \sigma^2 > 0$$
$$= 0, \qquad \sigma^2 < 0$$

Here g is the renormalized photon–neutrino weak coupling constant.

It is noted that here we come across the same divergence difficulties as in QEDP and these have to be removed by cutoffs. That is, apart from the dependence of Z_3 on the mass and coupling constant, Z_3 is also dependent on the cutoff factor, and we write in a generalized form

$$Z_3^{-1} = 1 + \int_0^{\Lambda^2} f(M^2, g, m_0)\frac{dM^2}{M^2}$$

where m_0 is the renormalized photon mass.

However, it is to be observed that for two-component neutrinos, the renormalized mass as well as the bare mass is zero. The vanishing of the physical and bare mass of neutrinos is ensured by the fact that the total lagrangian for the photon–neutrino weak interaction is invariant under the transformation

$$\psi \to \exp(i\alpha\gamma_5)\psi$$

which shows that the interaction term does not contribute to any mass effect. Considering that the physical mass of a neutrino is zero, we note that the renormalized photon mass m_c must be identically zero. This follows from the fact that if a photon composed of a pair consisting of a massless neutrino and antineutrino attains a certain mass due to interaction, it will be unstable and should decay spontaneously into a neutrino–antineutrino pair. However, this is in contradiction to the fact that a real photon having a rest mass, however small, cannot interact weakly with neutrinos. Indeed, this is because if a weak interaction of massive photons is allowed, we cannot forbid a priori a gauge-noninvariant interaction such as $ig\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_e A_\mu$ depicting the weak interaction of photons with electrons. But this leads to a contradiction, for in that case, photons in an external field should create electron–positron pairs both electromagnetically and weakly, which is absurd. So the weak interaction of a photon behaving as a particle of nonzero rest mass must be forbidden. Thus zero is essentially an isolated point of the spectrum of p_μ^2 for the photon field. This shows that in neutrinodynamics, the vanishing of Z_3 can arise from the functional dependence on the external parameters. Thus the main difficulty which crops up in using the condition in QEDP is removed in neutrinodynamics.

Now we note that the unique result $Z_1 = Z_2 = Z_3 = 0$ in neutrinodynamics does not allow us to write the Lagrangian using the unrenormalized field quantities. In this context here we calculate the neutrino self-energy and photon self-energy in QND.

(a) Neutrino Self-Energy. Here we calculate the second-order self-energy diagram in neutrinodynamics. The neutrino self-energy diagram is shown in Figure 1. According to the photon–neutrino weak coupling theory, the photon–neutrino weak interaction is given by the Lagrangian

$$L_I = ig\bar{\phi}_\nu\gamma_\mu\phi_\nu A_\mu \tag{6}$$

where g $(=10^{-10}e)$ is the photon–neutrino weak coupling constant (Bandyopadhyay, 1968) and ϕ_ν is the two-component spinor. In fact from the condition $Z_1 = Z_2 = Z_3 = 0$ and from the relations (1)–(4), we note that the unrenormalized field quantities $\phi_\nu^{(u)}$ and $A_\mu^{(u)}$ become zero so that the physical field quantities ϕ_ν and A_μ become finite. In fact, this suggests that

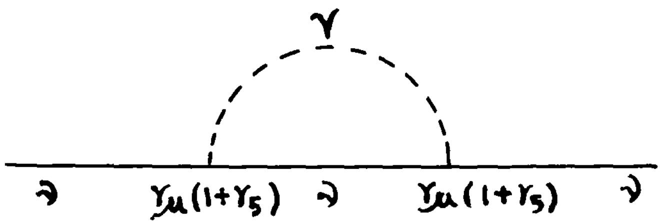

Fig. 1. Neutrino self-energy diagram of second order.

the Lagrangian cannot be expressed in terms of unrenormalized field quantities and so the term "renormalization" here bears a special meaning, indicating just the existence of normalized field quantities ϕ_ν and A_μ in the physical world. However, the condition $g = Z_1^{-1}Z_2Z_3^{1/2}g^{(u)}$ with $Z_1 = Z_2 = Z_3 = 0$ suggests that the unrenormalized coupling constant $g^{(u)}$ must be infinite to have the interaction Lagrangian (6) meaningful. Thus the existence of the Lagrangian (6) is determined simply by the condition $g^{(u)} = \infty$. Having considered this, we now transform ϕ_ν into a four-component spinor ψ_ν by the relation

$$\phi_\nu = \tfrac{1}{2}(1 + \gamma_5)\psi_\nu \tag{7}$$

the interaction Lagrangian can be written as

$$L_I = ig\bar{\psi}_\nu\gamma_\mu(1 + \gamma_5)\psi_\nu A_\mu \tag{8}$$

This Lagrangian is invariant under the transformation

$$\psi \to \exp(i\alpha\gamma_5)\psi \tag{9}$$

which suggests that the interaction term does not contribute to any mass effect. This suggests that the bare mass as well as the physical mass of a neutrino is zero. Considering $m^{(u)} = 0$ and $Z_3 = 0$ we can replace the Lagrangian in a more generalized form by expressing it term by term in perturbation theory:

$$L_I = \lim_{\epsilon \to 0} \sum Z(\epsilon^2)\frac{h(x, \epsilon\mathbf{n}^{(\mu)})_+^+}{2\epsilon} - (\text{vacuum expectation value}) \tag{10}$$

With

$$h(x, \epsilon\mathbf{n}^{(\mu)}) = i\bar{\psi}\,(x + \epsilon\mathbf{n}^{(\mu)})\gamma_\mu(1 + \gamma_5)\mathbf{n}\phi(x, \epsilon\mathbf{n}^{(\mu)})\psi(x - \epsilon\mathbf{n}^{(\mu)}) \tag{11}$$

and $\mathbf{n}^{(\mu)}$ being a unit vector in the direction of the μth axis, ϵ represents a bound on nonlocal interaction. Here the upper index of h indicates that it is symmetric under the two commuting operations of charge conjugation while the lower index indicates the Hermitian conjugation, i.e.,

$$h_+^+ = \tfrac{1}{4}(h + h^c + h^+ + h^{+c}) \tag{12}$$

ϕ is a spectral function which has the following property:

$$\phi^+(x, \epsilon\mathbf{n}) = \phi(x, -\epsilon\mathbf{n})$$

and

$$h^+(x, \epsilon\mathbf{n}) = -h(x, \epsilon\mathbf{n}) \tag{13}$$

Now, we calculate the self-energy diagram (Figure 1) in the conventional way. We write the contribution of the diagram by the expression

$$\Sigma(p) = \frac{ig^2}{(2\pi)^4} \int \gamma_\mu(1+\gamma_5)\frac{1}{\hat{p}-\hat{k}}\gamma_\mu(1+\gamma_5)\frac{-i}{k^2}\,d^4k \tag{14}$$

where $\hat{a} = \gamma_\mu a_\mu$. Following the usual procedures, we can reduce the expression for $\Sigma(p)$ to the form

$$\Sigma(p) = \frac{g^2}{4\pi^2}\left[\frac{\hat{p}}{4} + \frac{1}{i\pi^2}\int d^4k \int_0^1 \frac{\hat{p}(1-x)}{[k^2 + p^2x(1-x)]^2}\,dx \right. \tag{15}$$

It is here noted that the spectral representation of the neutrino propagator with vanishing bare and physical mass is given by

$$-\frac{1}{G^\nu(p)} = \frac{\hat{p}}{Z_1} \tag{16}$$

As considered earlier, if we replace the renormalization constant Z_1 by a gauge-dependent function $Z(\epsilon^2)$ where $\epsilon \to 0$, as discussed by Maris and Dillenburg (1970) we have $Z(0) = Z_1 = Z_2$. So in the limiting process we can write the expression (16) as

$$-\frac{1}{G^\nu(p)} = \left.\frac{\hat{p}}{Z(\epsilon^2)}\right|_{\epsilon \to 0}$$

This suggests that in the limit $\epsilon \to 0$, $p^2 \to 0$ for $G^\nu(p)$ to be finite.

This shows that for $Z_1 = 0$ p^2 must vanish even when the neutrino is in the virtual state. That is, in the virtual state the neutrino should be on the mass shell. This suggests that in the self-energy diagram (Figure 1), the four-momentum square of the virtual neutrino $(p-k)^2$ should tend to zero in the limiting process $\epsilon \to 0$. Now from the relations $p^2 \to 0$, $(p-k)^2 \to 0$ as $\epsilon \to 0$ where p is the four-momentum of the initial neutrino and k is the four-momentum of the virtual photon, we have the additional constraint that since the virtual neutrino is on the mass shell, $|\mathbf{k}| \leqslant k_0 < p_0$. Thus the expression (15) is found to take the form, putting $c^2 = x(1-x)$ and transforming the integral in the Euclidean space,

$$\frac{p^2}{\hat{p}}\int \frac{d^2k}{(k^2+p^2c^2)^2} \sim \frac{p^2}{\hat{p}}\int_0^\infty \frac{|k|^3\, d|k|}{(|k|^2+p^2c^2)^2}$$

which behaves as $p^2 \ln p_0^2 - p^2 \ln p^2$ because of the condition $|\mathbf{k}| \leqslant |k_0| < p_0$. So we see that in the limit $p^2 \to 0$, the expression (15) vanishes. Thus we get the interesting result that the self-energy diagram vanishes in neutrinodynamics.

It may be remarked here that the compositeness condition $Z_3 = 0$ actually suggests that any trilinear coupling (Yukawa type of coupling) can be reduced to a four-fermion Fermi coupling. But in the trilinear photon–neutrino weak coupling $ig\bar{\phi}_\nu \gamma_\mu \phi_\nu A_\mu = ig\bar{\psi}_\nu \gamma_\mu (1+\gamma_5)\psi_\nu A_\mu$ the coupling constant g is dimensionless, whereas when we reduce it to the four-fermion coupling of neutrino currents, the coupling constants have the dimension $(\text{mass})^{-2}$. However, in neutrinodynamics all the fields are massless and so the theory is dilatational invariant. Hence, the self-consistency of the theory requires that we must introduce the dimension of mass in the theory; otherwise when the interaction of the composite photon with the neutrino current is written in the form of a four-fermion Fermi coupling the theory becomes meaningless. So, to have a meaningful theory of neutrinodynamics, we must introduce the notion of fundamental length l_0 in nature.

The theory of QND has a dilatational symmetry and so the fundamental length should show up when the symmetry is broken. In a previous paper (Bandyopadhyay, 1973) we have shown that the mass (as well as charge) of a lepton can be achieved through a nonlocal field theoretic interaction of photons and neutrinos when symmetry is spontaneously broken. In this picture space-time quantized domain with a fundamental length is the seat of a charged and massive lepton. This helps us to have a unified theory of weak and electromagnetic interactions and the spontaneous breakdown of dilatational symmetry appears when a charged and massive lepton is generated through nonlocal interactions so that the dimension of a fundamental length l_0 is specified.

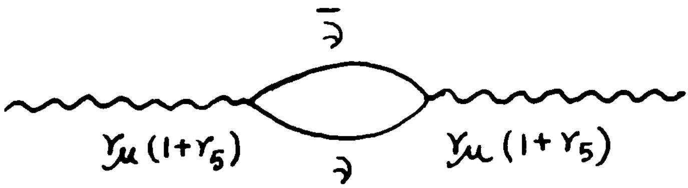

Fig. 2. Photon self-energy diagram of second order in neutrinodynamics.

(b) Photon Self-Energy. The scattering matrix in the perturbation expansion can be integrated by the use of the conventional Feynman rules and we can easily differentiate these from "ad hoc" rules generally used in specific calculation which are valid for all orders of perturbation expansion. Hence, on the basis of the aforesaid picture the gauge variant photon self-energy arises in second order. Now we consider the photon self-energy diagram in neutrinodynamics (Figure 2). Here the two-corner loop yields the symmetric tensor $\pi_{\mu\nu}(k)$ in momentum space which must be Lorentz invariant as well as gauge invariant. Now the insertion of a photon self-energy into an internal photon line has the effect of replacing the photon propagation function $D_0(k)$ by another functions $Z_3 D(k)$. So, for the second-order self-energy parts, the photon propagator takes the form

$$Z_3 D(k) = D_0(k) + D_0(k) g^2 \pi_{\mu\nu}(k) D(k) \tag{18}$$

and

$$\pi_{\mu\nu}(k) = \left(k_\mu k_\nu - k^2 g_{\mu\nu}\right) c(k^2) \tag{19}$$

These suggest that

$$\lim_{k \to 0} \pi_\mu^\mu(k) = 0 \tag{20}$$

In neutrinodynamics, for the compositeness condition $Z_3 = 0$ we have

$$D_{0\mu\nu}(k) = \left(\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right) k^{-2} \tag{21}$$

and that

$$g^2 \pi_\rho^\mu(k) D_{\rho\nu}(k) + \left(\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right) = 0 \tag{22}$$

which represents an equation for the photon propagator. The solution of equation (22) is

$$D_{\mu\nu}(k) = \left(\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right) g^{-2} c^{-1}(k^2) k^{-2} - a \cdot \frac{k_\mu k_\nu}{k^4} \tag{23}$$

Here the term $a \cdot k_\mu k_\nu / k^4$ comes out in the following manner. The solution of equation (22) contains an additive form $(k_\mu k_\nu / k^2) D_\mu(k)$ because this component is annihilated by a $(g_{\mu\nu} k^2 - k_\mu k_\nu)$ factor occurring in $\pi_{\mu\nu}(k)$. So, $a \cdot k_\mu k_\nu / k^4$ is a gauge-dependent factor, where a stands for the gauge parameter.

The second-order diagram contains at most vertices, each with a parameter ϵ which is finally brought to zero. If there are divergent diagrams the ϵ limits of different diagrams are not independent. The prescription is to number the ϵ's in each diagram to take the ϵ's with same number in different diagram equal to sum the diagrams, and to bring first ϵ_2 and finally ϵ_1 to zero. We now apply these rules to the second-order photon self-energy diagram. As a result this gives a quadratically divergent, Lorentz- and gauge-noncovariant contribution which cancels exactly the well-known quadratic divergence of the second-order diagram, resulting in a gauge- and Lorentz-invariant vacuum polarization. Hence, the photon propagator may be written in the form:

$$D_{\mu\nu}(k) = Z_1^{-1}(\epsilon^2) \pi_{\mu\nu}(k) \tag{24}$$

Now adopting the usual technique of Maris et al. (1968), we can write the expression of $\pi_{\mu\nu}(k)$ for the second-order photon self-energy in the following form:

$$\pi_{\mu\nu}(k) = \frac{ig^2}{(2\pi)^4} \lim_{\epsilon_1^\mu \to 0} \lim_{\epsilon_2^\mu \to 0} \int d^4p \cos[(2p-k)_\mu \epsilon_1^\mu] \cos[(2p-k)_\nu \epsilon_2^\nu]$$

$$\times \tilde{\delta}(k, \epsilon_1^\mu)\, \tilde{\delta}(k, \epsilon_2^\mu) \mathrm{Tr}\left[\gamma_\mu (1+\gamma_5) \frac{1}{\hat{p} - \hat{k}} \gamma_\mu (1+\gamma_5) \frac{1}{\hat{p}}\right] \tag{25}$$

where

$$\tilde{\delta}(k, \epsilon^\mu) = \frac{\sin k\epsilon^\mu}{k\epsilon^\mu} \tag{26}$$

After detailed calculation equation (25) reduces to the form

$$\pi_\mu^\mu(k) = \frac{ig^2}{(2\pi)^4} \lim_{\epsilon_1^\mu \to 0} \int d^4p \left[\tilde{\delta}(k, \epsilon_1^\mu)\right]^2$$

$$\times \cos\left[(2p-k)_\mu \epsilon_1^\mu\right] \frac{p^2 - p_1 k}{p^2 \cdot (p-k)^2} \tag{27}$$

The difficulty that now arises from equation (27) is that it does not lead to the equation (20), a violation of gauge invariance. However, the usual deduction of $\pi_{\mu\nu}(0) \neq 0$ from equation (27) is invalid because this deduction involves an unjustified interchange of the limit $k \to 0$ with p integration. If this interchange is carried out a confluence of the poles of the integrand results and the integral diverges. Moreover, the above result ensures a gauge-invariant equation (20). This can be seen as follows:

It is well known that the Feynman contour is equivalent to

$$\frac{1}{p^2 + \Lambda^2 - i\epsilon} \to P \frac{1}{p^2 + \Lambda^2} + i\pi\delta(p^2 + \Lambda^2) \tag{28}$$

where the integration is performed along the real p_0 axis and P denotes the Cauchy principal value. Using equation (28) in equation (27) and equating the real and imaginary parts we have

$$\mathrm{Re}\left[\pi_\mu^\mu(k)\right] = -\frac{g^2}{2\pi^3} \lim_{\epsilon_1^\mu \to 0} P \int d^4p\, d^4p' \cos\left[(2p-k)_\mu \epsilon_1^\mu\right]\left[\tilde{\delta}(k, \epsilon_1^\mu)\right]^2$$

$$\times \delta(p - p' - k)(p, p' + 2\Lambda^2)\left[\frac{\delta(p^2 + \Lambda^2)}{p'^2 + \Lambda^2} + \frac{\delta(p'^2 + \Lambda^2)}{p^2 + \Lambda^2}\right] \tag{29}$$

and

$$\mathrm{Im}\, \pi_\mu^\mu(k) = \frac{g^2}{2\pi^4} \lim_{\epsilon_1^\mu \to 0} \int d^2p\, d^4p' \delta(p - p' - k)(p, p' + 2\Lambda^2)\left[\tilde{\delta}(k, \epsilon_1^\mu\right]^2$$

$$\times \cos\left[(2p-k)_\mu, \epsilon_1^\mu\right]$$

$$\times \left[P \frac{1}{p^2 + \Lambda^2} P \frac{1}{p'^2 + \Lambda^2} - \pi^2 \delta(p^2 + \Lambda^2)\, \delta(p'^2 + \Lambda^2)\right] \tag{30}$$

This imaginary part is to vanish for $k = 0$. Now our object is to show that the real part is also a vanishing quantity; in order to derive the condition we know that

$$P\int f(x)\,\delta(x)\,dx = 0 \tag{31}$$

for all functions $f(x)$ which have at most a simple pole at $x = 0$. The Cauchy principal part requires an integral over an interval which excludes exactly the nonvanishing contribution of the δ function. Thus, the limit $k \to 0$ can here even be interchange with the integration, so

$$\begin{aligned}\operatorname{Re}\left[\pi_{\mu}^{\mu}(0)\right] &= -\frac{g^2}{\pi^3}\lim_{\epsilon_1^{\mu}\to 0} P\int d^4p\,(p^2+2\Lambda^2)\frac{\delta(p^2+\Lambda^2)}{p^2+\Lambda^2}\\ &= 0\end{aligned} \tag{32}$$

Thus the photon self-energy vanishes and the gauge-invariance requirement (20) is satisfied. Moreover, from equation (24) it is evident that $Z(\epsilon^2) = 0$, i.e., $Z_1 = Z_2 = 0$.

Hence we arrive at a very significant conclusion in QND that all the renormalization constant become zero and as such all the unrenormalized quantities disappear from the field equation. This special feature makes QND a finite field theory.

3. DISCUSSION

We have shown above that the theory of QND has the most interesting property that all the renormalization constants $Z_1 = Z_2 = Z_3 = 0$, which forbids the field equations to be expressed in terms of unrenormalized field quantities. This makes the conventional notion of renormalization here meaningless. This is a significant result in quantum field theory as we know that in perturbation theory of QED the renormalization constants are infinite so that each calculation of a physical quantity has an infinity buried in it. Thus the theory of quantum neutrinodynamics avoids the inconsistencies inherent in renormalization procedure and represents a quantum field theory devoid of divergences.

Again from the viewpoint of a unified model of weak and electromagnetic interaction, based on the concept of dynamical origin of charge and mass of an electron (Bandyopadhyay, 1973a), it becomes evident that QND is the basic field theory and an electron is created by n number of photon–neutrino weak interactions at different space-time points and the quantization of charge in units of e is related to the quantization of space-time (Bandyopadhyay, 1973, 1974). Since the basic field theory is QND, so the unified model of weak and electromagnetic interaction suggests that QED is also convergent as manifested in QND.

ACKNOWLEDGMENT

The authors are grateful to R. K. Roychoudhury for helpful discussions.

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